



Oxford Cambridge and RSA

**Thursday 23 June 2022 – Afternoon**

**A Level Further Mathematics B (MEI)**

**Y435/01 Extra Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 Three sequences,  $a_n$ ,  $b_n$  and  $c_n$ , are defined for  $n \geq 1$  by the following recurrence relations.

$$(a_{n+1} - 2)(2 - a_n) = 3 \text{ with } a_1 = 3$$

$$b_{n+1} = -\frac{1}{2}b_n + 3 \text{ with } b_1 = 1.5$$

$$c_{n+1} - \frac{c_n^2}{n} = 1 \text{ with } c_1 = 2.5$$

The output from a spreadsheet which presents the first 10 terms of  $a_n$ ,  $b_n$  and  $c_n$ , is shown below.

	A	B	C	D
1	$n$	$a_n$	$b_n$	$c_n$
2	1	3	1.5	2.5
3	2	-1	2.25	7.25
4	3	3	1.875	27.28125
5	4	-1	2.0625	249.0889
6	5	3	1.96875	15512.32
7	6	-1	2.01563	48126390
8	7	3	1.99219	3.86E+14
9	8	-1	2.00391	2.13E+28
10	9	3	1.99805	5.66E+55
11	10	-1	2.00098	3.6E+110

**Without** attempting to solve any recurrence relations, describe the apparent behaviour, including as  $n \rightarrow \infty$ , of

- $a_n$
- $b_n$
- $c_n$

[7]

2 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 10 & 12 & -8 \\ -1 & 2 & 4 \\ 3 & 6 & 2 \end{pmatrix}$ .

(a) In this question you must show detailed reasoning.

Show that the characteristic equation of  $\mathbf{A}$  is  $-\lambda^3 + 14\lambda^2 - 56\lambda + 64 = 0$ . [3]

(b) Use the Cayley-Hamilton theorem to determine  $\mathbf{A}^{-1}$ . [5]

A matrix  $\mathbf{E}$  and a diagonal matrix  $\mathbf{D}$  are such that  $\mathbf{A} = \mathbf{EDE}^{-1}$ . The elements in the diagonal of  $\mathbf{D}$  increase from top left to bottom right.

(c) Determine the matrix  $\mathbf{D}$ . [4]

3 A sequence is defined by the recurrence relation  $5t_{n+1} - 4t_n = 3n^2 + 28n + 6$ , for  $n \geq 0$ , with  $t_0 = 7$ .

(a) Find an expression for  $t_n$  in terms of  $n$ . [6]

Another sequence is defined by  $v_n = \frac{t_n}{n^m}$ , for  $n \geq 1$ , where  $m$  is a constant.

(b) In each of the following cases determine  $\lim_{n \rightarrow \infty} v_n$ , if it exists, or show that the sequence is divergent.

(i)  $m = 3$  [1]

(ii)  $m = 2$  [1]

(iii)  $m = 1$  [1]

4 A binary operation,  $\circ$ , is defined on a set of numbers,  $A$ , in the following way.

$$a \circ b = k_1 a - k_2 b + k_3, \text{ for } a, b \in A,$$

where  $k_1, k_2$  and  $k_3$  are constants (which are not necessarily in  $A$ ) and the operations addition, subtraction and multiplication of numbers have their usual notation and meaning.

You are initially given the following information about  $\circ$  and  $A$ .

- $A = \mathbb{R}$
- $0 \circ 0 = 2$
- An identity element,  $e$ , exists for  $\circ$  in  $A$

- (a) Show that  $a \circ b = a + b + 2$ . [5]
- (b) State the value of  $e$ . [1]
- (c) Explain whether  $\circ$  is commutative over  $A$ . [1]
- (d) Determine whether or not  $(A, \circ)$  is a group. [6]
- (e) Explain whether your answer to part (d) would change in each of the following cases, giving details of any change.
- (i)  $A = \mathbb{Z}$  [1]
- (ii)  $A = \{2m : m \in \mathbb{Z}\}$  [1]
- (iii)  $A = \{n : n \in \mathbb{Z}, n \geq -2\}$  [1]

5 A surface  $S$  is defined by  $z = f(x, y)$ , where  $f(x, y) = ye^{-(x^2+2x+2)y}$ .

(a) (i) Find  $\frac{\partial f}{\partial x}$ . [1]

(ii) Show that  $\frac{\partial f}{\partial y} = -(x^2y + 2xy + 2y - 1)e^{-(x^2+2x+2)y}$ . [1]

(iii) Determine the coordinates of any stationary points on  $S$ . [4]

Fig. 5.1 shows the graph of  $z = e^{-x^2}$  and Fig. 5.2 shows the contour of  $S$  defined by  $z = 0.25$ .

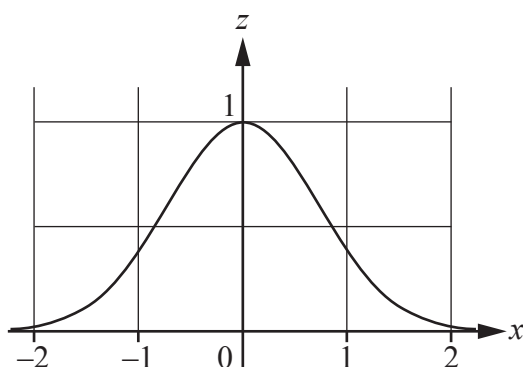


Fig. 5.1

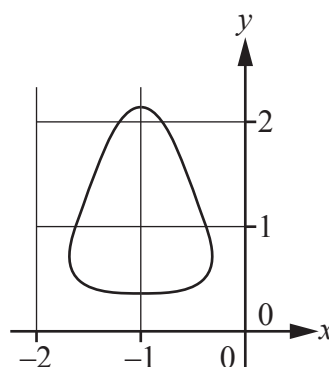


Fig. 5.2

(b) Specify a sequence of transformations which transforms the graph of  $z = e^{-x^2}$  onto the graph of the section defined by  $z = f(x, 1)$ . [2]

(c) Hence, or otherwise, sketch the section defined by  $z = f(x, 1)$ . [1]

(d) Using Fig. 5.2 and your answer to part (c), classify any stationary points on  $S$ , justifying your answer. [2]

You are given that  $P$  is a point on  $S$  where  $z = 0$ .

(e) Find, in vector form, the equation of the tangent plane to  $S$  at  $P$ . [4]

The tangent plane found in part (e) intersects  $S$  in a straight line,  $L$ .

(f) Write down, in vector form, the equation of  $L$ . [1]

**END OF QUESTION PAPER**

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